- In general,

$$y_{n-k} = (1-\nabla)^k y_n$$

$$= \left(1 - {}^kC_1 \nabla + {}^kC_2 \nabla^2 - {}^kC_3 \nabla^3 + \cdots + (-1)^k \nabla^k\right) y_n$$

$$\therefore \quad y_{n-k} = y_n - {}^kC_1 \nabla y_n + {}^kC_2 \nabla^2 y_n - {}^kC_3 \nabla^3 y_n + \cdots + (-1)^k \nabla^k y_n$$

Pb

1. Find $y(-1)$ if $y(0)=2$, $y(1)=9$, $y(2)=28$, $y(3)=65$, $y(4)=126$ & $y(5)=217$

sol.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0	2				
1	9	7			
2	28	19	12		
3	65	37	18	6	
4	126	61	24	6	0
5	217	91	30	6	0

$$y(-1) = y_{-1} = y_{5-6}$$

$$= y_5 - {}^6C_1 \nabla y_5 + {}^6C_2 \nabla^2 y_5 - {}^6C_3 \nabla^3 y_5 + {}^6C_4 \nabla^4 y_5 - \cdots$$

$$= 217 - 6(91) + 15(30) - 10(6) + 0$$

$$= 667 - 666$$

$$\boxed{y(-1) = 1}$$

2. Given $y_3 = 2$, $y_4 = -6$, $y_5 = 8$, $y_6 = 9$ and $y_7 = 17$, Calculate $\Delta^4 y_3$.

sol

$$\Delta^4 y_3 = (E-1)^4 y_3$$

$$= (E^4 - 4E^3 + 6E^2 - 4E + 1) y_3$$

$$= E^4 y_3 - 4E^3 y_3 + 6E^2 y_3 - 4E y_3 + y_3$$

$$= y_7 - 4y_6 + 6y_5 - 4y_4 + y_3$$

$$= 17 - 4(9) + 6(8) - 4(-6) + 2$$

$$\boxed{\Delta^4 y_3 = 55}$$

$\because E^n y = y_{x+nh}$

$E^n f(x) = f(x+nh)$.

3. Find y_6 if $y_0 = 9$, $y_1 = 18$, $y_2 = 20$, $y_3 = 24$ given that the third differences are constants.

Sol: Since third differences are constants,

$$\Delta^4 y_0 = \Delta^5 y_0 = \Delta^6 y_0 = 0$$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+(-b))^3 =$

$\binom{n}{1} = {}^nC_1 = n$

$\binom{n}{n} = {}^nC_n = 1$

$\binom{n}{0} = {}^nC_0 = 1$

$0! = 1$

$$y_6 = E^6 y_0 = (1+\Delta)^6 y_0$$

$$= (1 + 6C_1\,\Delta + 6C_2\,\Delta^2 + 6C_3\,\Delta^3 + \dots \Delta^6) y_0$$

$$= (1 + 6\Delta + 15\Delta^2 + 20\Delta^3) y_0 \quad \text{Since other term vanish.}$$

$$= [1 + 6(E-1) + 15(E-1)^2 + 20(E-1)^3]\, y_0$$

$$= (1 + 6E - 6 + 15E^2 - 30E + 15 + 20E^3 - 60E^2 + 60E - 20) y_0$$

$$= (-10 + 36E - 45E^2 + 20E^3) y_0$$

$$= -10y_0 + 36y_1 - 45y_2 + 20y_3$$

$$= -10(9) + 36(18) - 45(20) + 20(24)$$

$$= 138.$$

$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \binom{n}{2}x^{n-2}a^2 + \dots + \binom{n}{n}x^0 a^n$

$(a+b)^4 = a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}ab^3 + b^4$

$= a^4 + 4a^3b + \frac{4\times 3}{2\times 1}a^2b^2 + 4ab^3 + b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

${}^{100}C_{95} = {}^{100}C_5$

${}^nC_r = {}^nC_{n-r}$

$(a+b)^4 = (a+b)^2 (a+b)^2$

$(a^2+b^2+2ab)(a^2+b^2+2ab)$

$a^4 + a^2b^2 + 2a^3b + b^2a^2 + b^4 + 2ab^3$

$+ 2a^3b + 2ab^3 + 4a^2b^2$

$a^4 + 6a^2b^2 + 4a^3b + 4ab^3 + b^4$